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Nuclear and General Purpose Forces**

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# STABILITY OF NUCLEAR AND GENERAL PURPOSE FORCES

Gregory H. Canavan

The analysis of the crisis stability is extended to forces that contain both nuclear and conventional elements through the definition of the rough nuclear equivalents. The variation in overall stability indices is dominated by the reduction in conventional second strikes. The stability of roughly equal-force configurations is neutral for small nuclear kill probabilities, but falls as their kill probability increases unless supplemented by survivable or ex-theater forces.

**Introduction.** Companion papers derived a model for the crisis stability of interactions between two nuclear forces. This note extends that analysis to the stability of exchanges between forces containing both nuclear and conventional elements. It uses aggregated, probabilistic models for their exchanges, and conventional equations for the magnitudes of strikes, costs, and stability indices. The model is used to study stability at force levels smaller than the number of value targets held at risk, in which case costs, indices, and optimal allocations are linear in the forces, and the sources of various trends is clear. The equations for a model with adequate fidelity are solved analytically by minimizing first strike costs. These analytic optimizations agree closely with numerical solutions.

The discussion treats the interaction between two forces, of which one has a significant nuclear capability and the other has conventional superiority. The key parameters are the effectiveness of the nuclear weapons against dispersed conventional forces and the effectiveness of conventional forces against nuclear bases, which are studied in detail. Extended conflicts are reduced to the effective first and second strikes each side could deliver through the definition of the rough nuclear equivalence of conventional forces.

Optimal allocations of nuclear weapons against conventional forces are proportional to the size of those conventional forces, although, they saturate for large forces, where all weapons are applied to military value targets to limit damage. Conventional force allocations to nuclear forces fall as conventional forces increase, reflecting a shift from damage limiting to simply damaging. For large kill probabilities, the allocation of nuclear weapons to military forces falls because fewer weapons are required for suppression, so it is more advantageous to destroy value. As the nuclear kill probability falls, the allocation reaches a maximum and then falls sharply as weapons become ineffective against dispersed forces. As forces might be dispersed after commencement of hostilities, this implies some urgency to use nuclear weapons before they lose effectiveness, which is destabilizing.

The nuclear side's first strikes are a minimum at intermediate values of kill probability, where allocation to conventional forces are largest, and approach conventional levels for large

values of kill probability, where the survivability and second strike of conventional forces is reduced. The difference between the nuclear side's second and first strikes are close to zero for intermediate and large values kill probabilities where the difference between the conventional side's second and first strikes fall due to the reduction in his second strikes.

The stability of the roughly equal-force configuration studied here is neutral for small nuclear kill probabilities, but falls as the kill probability increases—to significantly reduced levels when nuclear weapons are fully effective. The variation in overall stability indices is dominated by the reduction in the conventional second strike for kill probabilities at which the nuclear strikes are fully effective.

Decreasing the effectiveness of conventional forces against weapons increases the nuclear side's and overall stability. Thus, if the conventional forces have few precision or deep-strike weapons or if the nuclear weapons are well dispersed, stability is increased. However, the fundamental problem is that stability falls as the nuclear kill probability increases, i.e., as the nuclear forces make a greater contribution. That means introducing nuclear forces into a conventional engagement or making them more effective is destabilizing because they suppress the conventional side's second strike. Given the impact nuclear forces could have on undispersed conventional forces, the conventional force commander has a great incentive to disperse them through attack, which would start the war, or dispersal, which would reduce the effectiveness of the nuclear forces, giving the nuclear forces an incentive to strike first.

**Exchange Model.** The model used is an aggregated, probabilistic exchange between the two forces that treats the nuclear forces as in earlier papers and the conventional forces in terms of nuclear force equivalents. That is, nuclear forces are treated as if they were  $M$  missiles with  $m$  weapons each, although the weapons might be mounted on aircraft instead; what matters is the total number of weapons,  $mM$ .  $M$  can represent either the number of missiles, missile sites, or the number of bases on which the missiles or aircraft are based by appropriate choice of  $m$  and  $M$ . Survivability of in-theater missiles and aircraft is an important feature, which is studied by varying the kill probability of opposing forces.

The opposing side, denoted by primes for accord with the symbols  $m'$  and  $M'$  used for its forces, is taken to have only conventional forces of magnitude  $m'M'$  in missile equivalents. There are simple models for conventional forces, which are controversial, and there are less controversial models, which are not simple. This treatment interprets  $m'M'$  as prime's excess of conventional force over unprime's conventional forces. That is prime's unattrited forces could seize or destroy  $m'M'$  targets.  $M'$  represents the number of points that would have to be targeted to destroy one of these force units, each of which has a value killing potential of  $m'$  targets. Survivability of conventional forces is achieved by dispersal, which is studied below by degrading prime's kill probability to model varying levels of dispersal.

The intent is not to develop a model of conventional war, but to evaluate the extent to which the models developed earlier for nuclear war can be applied in a logical fashion to treat the stability of engagements involving forces that have both conventional and nuclear weapons. The analysis below does that, maintaining the symbols used for both sides in the earlier analyses in order to simplify the drawing of the relevant analogies between the nuclear and mixed forces. In order to make the analysis concrete, it is necessary to relate the effectiveness of conventional forces to nuclear ones.

To do so, it is only necessary to observe that both types of forces can be used for both types of applications: negating opposing forces or destroying opposing value. That is, nuclear weapons can either be used to destroy conventional forces or to destroy the military value, airbases, forts, ports, command and control assets, that the conventional force commander could use to recover and re-attack after the strike. Similarly, conventional weapons can either be used to attack the bases or launchers on which the nuclear weapons are deployed or to seize or destroy value targets that the nuclear force commander could use to recover and re-attack.

These applications are not quite symmetrical. Nuclear forces can primarily be used to destroy forces or value targets; conventional forces can be used either to destroy or occupy military or value targets. For this study, the distinction is not essential, and occupation is taken to be tantamount to destruction. A second distinction is that nuclear strikes generally take place quickly, while conventional campaigns occupy days, weeks, or months. That distinction is reduced here by assuming that the conventional attack could take place quickly and that occupation is equivalent to destruction. Then the exchanges are roughly symmetric.

It is still necessary to relate the effectiveness of nuclear and conventional forces. The sensitivity of stability indices to the number of missiles (or airbases)  $M$  and the number of nuclear weapons per missile (or airbase)  $m$  has been assessed in earlier papers; thus, it is simplest to recast conventional forces in these terms. Conventional forces can be idealized in terms of the number of maneuver units in the theater. Undispersed battalions or brigades might occupy territories about as large as those that could be destroyed by one or a few tactical nuclear weapons. Thus, there is a rough equivalence between these units and the number of missile sites  $M'$  used in the strategic analyses.

The second parameter is the number of military or value targets that each of these units could address. If a maneuver unit had  $m'$  rockets, it could attack a like number of airbases, or a like number of cities. Thus,  $m'$  corresponds roughly to the number of weapons per missile of the strategic analyses. The calculations below use a nominal confrontation in which there are  $M = 5$  missiles or airbases with 10 weapons each for a total of 50 nuclear weapons facing a conventional force of 20 maneuver units with an attack potential of  $m' = 3$  units each for a total of 60 weapon equivalents. That is, the conventional forces have a slight advantage in numbers

and a three-fold advantage in distribution of weapons over units—the equivalent of deMIRVing. The initial calculations assume a kill probability of 0.6 for both forces. The conventional force has an additional advantage in its ability to disperse, which reduces the kill probability of the nuclear forces significantly.

**Exchange equations.** It is possible to model exchanges between these forces in terms of the first, F, and second, S, strikes prime could deliver to unprime and the first, F', and second, S', strikes unprime could deliver to prime.<sup>1</sup> If unprime strikes first and a fraction f of his weapons is directed at prime's vulnerable missiles, his first strike on prime's value targets is

$$F = (1 - f)mM. \quad (1)$$

The average number of weapons delivered on each of prime's vulnerable units is

$$r = fmM/M'. \quad (2)$$

For r large, the average probability of survival of a prime target is<sup>2</sup>

$$Q' = q^r e^{-fW \ln q/M'}, \quad (3)$$

where  $p = 1 - q$ , is the attacking missile's single shot probability of kill. Prime's second strike is

$$S' = m'M'Q' - m'M'q^r, \quad (4)$$

which is used to strike or seize value, as forces remaining at the end of the exchange are taken to have no value. The corresponding equations for prime's first strike can be derived either by repeating the logic from his perspective or simply by conjugating the equations above, i.e., interchanging primed and unprime symbols in Eq. (1)

$$F' = (1 - f')m'M'. \quad (5)$$

Similarly, unprime's second strike is given by conjugation as

$$S = mMQ - mMq'^r, \quad (6)$$

**Costs.** First and second strikes are converted into first and second strike costs through exponential approximations to the fractions of military value targets destroyed. The cost to unprime for striking first is approximated by

$$C_1 = (1 - e^{-kS'} + Le^{-k'F})/(1 + L), \quad (7)$$

where  $1/k' \approx 100$  is the number of prime military value targets unprime holds at risk,  $1/k \approx 100$  is the number of unprime military value targets prime holds at risk,<sup>3</sup> and L is a parameter that characterizes unprime's relative preference for damage to prime and preventing damage to self.<sup>4</sup> While these two costs are formally incommensurate, as they represent damage to different sides, it is conventional to approximate a total cost by taking their sum as a weighted sum with a parameter L, which is plausible but not unique.<sup>5</sup> For relatively small strikes,

$$(1 + L) C_1 \approx kS' + L(1 - k'F), \quad (8)$$

which is small. The cost to unprime for striking second is

$$(1 + L) C_2 = 1 - Le^{-kF'} + Le^{-k'S} \sim kF' + L(1 - k'S). \quad (9)$$

First and second strike costs for prime can be obtained either by re-deriving these results from prime's viewpoint or by conjugating Eqs. (7) and (9), which introduces a second constant  $L'$ , reflecting prime's relative attack preference<sup>6</sup>

**Indices.** While there is some arbitrariness in converting  $C_1$  and  $C_2$  into stability indices,<sup>7</sup> it is conventional to use the ratio of first and second strike costs,  $I = C_1/C_2$ , as a stability index for unprime, and  $I' = C_1'/C_2'$ , as a stability index for prime. When they are large, there is no advantage to striking first. When they are small, there is an apparent advantage, which may be perceived as an incentive to attack first in a crisis. For unequal forces, the product of the stability indices of the two sides is used as a compound index

$$\text{Index} = I \times I' = (C_1/C_2)(C_1'/C_2'). \quad (10)$$

For small  $F$  and  $S$ , the stability index is approximately<sup>8</sup>

$$I = C_1/C_2 \approx 1 + (C_1 - C_2)(1 + L)/L, \quad (11)$$

so an appropriate stability index for unprime for small forces is

$$J = (I - 1)L = (1 + L) (C_1 - C_2) \approx k(S' - kF') + Lk'(S - F), \quad (12)$$

for which  $J = 0$  represents neutral stability. It is clear from the appearance of  $S'$  and  $F'$  in this equation that the stability as seen by unprime depends on factors that will influence prime's perception of stability as well. By conjugation,

$$J' = k'(S - F) + L'k(S' - F'). \quad (13)$$

Their joint difference stability index is

$$\begin{aligned} J_t = J + J' &= k(S' - F') + Lk'(S - F) + k'(S - F) + L'k(S' - F') \\ &= k'(1 + L) (S - F) + k(1 + L') (S' - F'). \end{aligned} \quad (14)$$

For the  $k = k' = 1/100$  and  $L = L' = 0.5$  example treated here, this reduces to

$$J_{t,\text{symm}} = 2k(1 + L) [(S - F) + (S' - F')], \quad (15)$$

however, even for  $k = k'$  and  $L = L'$ ,  $S - F \neq S' - F'$ .

**Optimal attack allocation** for unprime amounts to choosing  $f$  that minimizes his first strike cost  $C_1$ ,<sup>9</sup> which can be accomplished in one of two ways. The first is by solving Eq. (7) for the value of  $f$  that minimizes  $C_1$ . While for large forces the equation is transcendental, it can be solved simply by iteration. The second way is by differentiating Eq. (7) with respect to  $f$ , setting the result to zero, and solving for  $f$ . For small forces ( $F, S \ll 1/k$ ) for which Eq. (7) reduces to Eq. (8), and it to

$$(1 + L) C_1 \approx km'M'e^{fmM} \ln q/M' + L[1 - k'(1 - f)mM], \quad (16)$$

which is sufficiently accurate for moderate forces.<sup>10</sup> Its derivative with respect to  $f$  has a minimum at

$$f_0 = (M'/mM \ln q) \ln(-Lk'/km' \ln q). \quad (17)$$

The equation for prime's first strike allocation is the conjugate of Eq. (14)

$$f_0' = (M/m'M' \ln q') \ln(-L'k/k'm \ln q'). \quad (18)$$

Figure 1 compares the numerical (n) and analytic allocations for forces with  $m = 10$  weapons on each of  $M = 5$  bases and  $m' = 3$  value target capacity forces on numbers of target ranging from  $M' = 5$  to 45 for  $p = p' = 0.6$ . As expected,  $f$  and  $f_n$  increase  $\sim M' < 25$ , where they reach unity and saturate with all weapons applied to military targets in an attempt to reduce damage as much as possible against strong odds. There is only a slight difference between the numerical and analytic allocations even for  $kM = 0.5$ . Conversely,  $f'$  and  $f_n'$  fall as  $M'$  increases, in accord with Eq. (17), reflecting the shift from damage to value to damage limitation against unprime's missiles as prime's force ratio increases.

Unprime's optimal  $f_0$  scales directly on prime's vulnerable targets  $M'$  and inversely on unprime's total weapons  $mM$ ; thus, if the number of vulnerable targets on each sides change proportionally,  $f_0 \sim 1/m$ , and to first order, prime's target kill capacity per target per missile determines unprime's optimal weapon allocation. If  $m$  does not change,  $f_0$  is constant.

Figure 2 shows the variation of the optimal allocation  $f$  with the kill probability of unprime's forces,  $p$ , for two values of unprime weapon effectiveness against forces,  $m' = 3$  and 6. The two top curves show  $f$  for  $m' = 6$  and 3, which are similar in shape. For  $m' = 3$ ,  $M' = 20$ , and  $p = 0.6$ , unprime's optimal allocation is 0.75 as in Fig. 1. For larger  $p$ , unprime's allocation to forces falls because fewer weapons are required to suppress them and it is more advantageous to destroy value. As  $p$  falls, the allocation reaches a peak of 0.85 at  $p = 0.4$ . For smaller  $p$ ,  $f$  falls sharply as weapons become ineffective against forces and hence are reallocated to more valuable prime targets. The decrease of  $f$  for small  $p$  represents the effectiveness of dispersal or concealment as a counter to nuclear weapons. As forces might be dispersed quickly after the commencement of hostilities, this implies some urgency to use those weapons before they lose effectiveness except in retaliation.

The two straight curves show  $f'$  for  $m' = 3$  and 6, where the higher effectiveness goes with the lower allocation, in accord with the scaling of Eq. (17),  $f_0' = 1/m'M'$ . Prime's optimal allocation is independent of unprime's effectiveness  $p$ .

**Optimal strikes.** With optimal allocation, the average number of weapons per target is

$$r = fmM/M' = \ln(-Lk'/km' \ln q)/\ln q, \quad (19)$$

so the survival probability is

$$Q' = q^r = -Lk'/km' \ln q. \quad (20)$$

Thus,  $r$  and  $Q'$  are largely determined by  $L/m'$  and  $q$ , the ratio of unprime's preference for damage to unprime's conventional effectiveness and unprime's kill probability. With these choices, unprime's first strike is

$$F = (1 - f_0)mM = mM - (M'/\ln q) \ln(-Lk'/km' \ln q); \quad (21)$$

and his second strike is

$$S = QmM = (-L'k/k' m \ln q')mM = -ML'k/k' \ln q', \quad (22)$$

which is independent of the number of missiles per base, but depends on prime's damage preference, the ratio of prime and unprime targets, and the effectiveness of prime's forces. The difference between them is

$$S - F = M[-L'k/k' \ln q' - m - (M'/M \ln q) \ln(-Lk'/km' \ln q)], \quad (23)$$

which indicates that for a fixed ratio of targets,  $M'/M$ ,  $S - F$ , and hence stability, depend on the bilinear product of  $M$  and a function of  $L$ ,  $k/k'$ ,  $m$ ,  $m'$ ,  $q$ , and  $q'$ . Prime's stability adds only  $L'$ . If survivable weapons or ex-theater forces were included, they would cancel out, as the number of such weapons delivered is the same in first and second strikes.<sup>11</sup>

Figure 3 shows the variation of each sides' first strikes as functions of unprime's kill probability  $p$  for the prime effectivenesses  $m' = 3$  and  $6$  of Fig. 2. The prime first strikes do not vary with  $p$  and increase in proportion to  $m'$ . The unprime first strikes drop at intermediate values of  $p$  because they are complementary to the  $f$  of Fig. 2. Unprime's first strike approaches prime's for  $p$  large, falls far below it at intermediate  $p$ .

Figure 4 shows the variation of each sides' survival probabilities as functions of unprime's kill probability  $p$  for the prime  $m' = 3$  and  $6$  of Fig. 2. The two top curves for  $Q'$  lie relatively close to one another, in accord with Eq. (20), which indicates that it should scale as  $Q' \sim -1/m' \ln q$ . The two bottom curves for  $Q$  lie on top of one another, in accord with Eq. (20)'s indication that they should scale as  $Q \sim -1/m \ln q'$ , in which neither  $m$  nor  $q'$  varies.

Figure 5 shows the variation of each sides' second strikes as functions of  $p$  for  $m' = 3$  and  $6$ , which closely follow the survival probabilities of the previous figure. Prime's  $S'$  curves drop sharply to roughly the same values as those for  $S$  by  $p \sim 0.9$ . The curves for  $S$  are constant throughout at the level  $mMQ = -mML'k/k' m \ln q' = -ML'/\ln q' = -20 \times 0.5 / \ln(0.4)$ <sup>11</sup>, set by the optimal solution, as neither  $M$  nor  $q'$  is varied.

Figure 6 shows the differences between the second and first strikes,  $S - F$  and  $S' - F'$ , as functions of  $p$  for  $m' = 3$  and  $6$ . The two curves for  $S - F$  lie close to the axis for  $0.2 < p < 0.7$ . For values of  $p$  outside that range their values tend to fall below zero. The curves for  $S' - F'$  fall as  $p$  increases due to the reduction in the second strikes in Fig. 5, i.e., as unprime's kill probability increases.

**Stability indices.** Figure 7 shows the variation of stability indices as functions of unprime's kill probability  $p$ . The top curve corresponds to the prime effectiveness  $m' = 3$  of Fig. 2. It is about zero for  $p = 0.1$ , which denotes neutral stability. It falls slowly to  $p = 0.2$ , and then falls more rapidly, reaching  $\sim -0.4$  by  $p = 0.9$ . The curve is surprisingly smooth, given the stronger behavior of the component curves on Fig. 6. In particular, the null behavior of  $J$  at  $p$  small results from a cancellation between  $S - F$ , which is decreasing strongly, and  $S' - F'$ , which is increasing. That means the nominal overall stability results from the averaging of the degrading stability seen by unprime with the increasing stability seen by prime.

The lower curve on Fig. 7 for  $m' = 6$  also starts near zero, but falls rapidly, reaching about  $-1$  at  $p = 0.9$ . Figure 6 shows that its null behavior at  $p = 0.1$  does not result from a cancellation; both curves are about zero there. Its behavior for larger  $p$  is dominated by that of  $S' - F'$ , as  $S - F$  is relatively small for  $m' = 6$  due to the saturation of  $f$  in Fig. 3. The curve for  $J'$  follows that for  $S' - F'$  closely out to about  $p = 0.6$ , after which  $S - F$  makes an additional small destabilizing contribution.

**Interpretation.** The overall impression is that stability decreases monotonically with increasing forces and with increasing force effectiveness  $m'$ . However, the actual variation is somewhat more complex, as shown by the cancellation between the instability seen by unprime and the stability seen by prime at small  $p$  for  $m' = 3$ —and the disappearance of that cancellation for force effectiveness only a factor of two larger.

There is one elementary variation; the effectiveness of prime's forces against unprime's weapons can be summarized simply: decreasing  $p'$  from the nominal value of 0.4 used above to 0.1 increases unprime and overall stability by about 0.2 units. Figure 8 shows that the effect of this change is essentially an upward translation of both curves by about this amount. A lower value of  $p'$  is perhaps appropriate if the conventional forces have few precision or deep strike weapons or if the nuclear weapons are well dispersed—although that may not be likely in theaters. If weapons from outside the theater were to be used, they might be characterized by very low values of  $p'$ , in which case the nuclear forces could be stabilizing.

The fundamental problem is that the stability curves fall as  $p$  increases, i.e., as the nuclear forces make a greater contribution, which indicates that the effect of introducing nuclear forces into a conventional engagement is destabilizing. The reason for that is shown in Fig. 2. It is that the curves for  $S' - F'$  both fall strongly as nuclear forces become more capable. That is because they suppress prime's second strike, as shown in Fig. 5, because of the reduced survival probabilities shown in Fig. 4. Given the great degradation that nuclear forces could inflict on conventional forces if they were struck before dispersed, the conventional force commander has a great incentive to disperse them over the battlefield. If he does that through attacking, that

starts the war. If he does it by dispersal and concealment, that threatens the effectiveness of the nuclear forces, which gives unprime and incentive to start the war. Both are destabilizing.

**Current applications.** The above results roughly model stability concerns in current theaters. For a number of decades, the US has used the threat of use of nuclear weapons in Europe as an offset to NATO's numerical inferiority in conventional forces. As shown above, the threat of use of capable nuclear weapons is destabilizing. Given the extensive preparation of Soviet forces, it appeared that they could also be ineffective. Both issues were addressed by linking the use of nuclear weapons in the theater to US strategic nuclear forces. That might not address the utility issue, but it did address the stability issue. By invoking out of theater forces, this step effectively decreased  $p'$  to very low levels, which translated the stability curves up to an extent such that the configuration should be stable for any value of  $p$ .

While the above explanation is not unambiguous, recent events have provided an additional perspective on this theater and these issues. After the Gulf War, Russia decided that the qualitative improvement in US forces meant that the conventional balance in Europe now favored NATO. Thus, Russia has decided to shift to greater reliance on nuclear weapons to stabilize the theater. In order to understand the impact of this shift, it is only necessary to interchange the roles of the two sides above. This reversal of Russian attitudes has essentially brought about an actual conjugation of the sides and forces that has given both meaning and utility to the conjugation process used to derive equations simply above.

**Summary and conclusions.** The model derived for nuclear missile exchanges is used to describe the interaction between two forces, of which one has a significant nuclear capability and the other has conventional superiority, which is treated in terms of nuclear force equivalents. The key parameters are the effectiveness of the nuclear weapons against dispersed conventional forces and the effectiveness of conventional forces against nuclear bases, both of which are modeled in detail. Survivability of in-theater missiles and aircraft is an important feature that is studied by varying the kill probability of opposing forces. Extended conflicts are modeled in terms of the effective first and second strikes each side could deliver to the other. The equations for a model with adequate fidelity are solved analytically for exchanges, costs, and stability indices by minimizing the cost of executing their first strikes. This analytic optimization agrees closely with numerical solutions.

Nuclear weapon allocations to conventional forces are proportional to the size of those forces, although for large forces, they saturate with all weapons applied to military value targets in an attempt to damage limit as much as possible. Conversely, conventional allocations to nuclear forces fall as the conventional forces increase, reflecting a shift from damage limiting to doing as much damage as possible with the forces that survive nuclear attack. The kill probability of nuclear weapons degrades against dispersed forces. For large kill probabilities,

their allocation to military forces falls because fewer weapons are required to suppress them, and it is more advantageous to destroy value. As the kill probability falls, the allocation reaches a maximum and then falls sharply as weapons become ineffective against dispersed forces and are reallocated to value targets. As forces might be dispersed after the commencement of hostilities, this implies some urgency to use nuclear weapons before they lose effectiveness except in retaliation, which is destabilizing.

Each sides' first strikes are complementary to these allocations, so the nuclear side's first strikes drop at intermediate values of  $p$  and approaches the conventional side's strikes for large values of their kill probability. As kill probability increases, the survivability and second strike of the conventional forces is reduced sharply. The difference between the nuclear side's second and first strikes are close to zero for intermediate and large values of their kill probability. The difference between the conventional side's second and first strikes fall as the nuclear side's kill probability increases due to the reduction in the conventional side's second strikes as the nuclear weapon's kill probability increases.

The stability of the roughly equal-force configuration is roughly neutral for small nuclear kill probabilities. It first falls slowly at the kill probability increases, then more rapidly, reaching significantly reduced levels when nuclear weapons are fully effective. The variation is smooth, given the stronger behavior of its components, but the stability indicated at small nuclear kill probabilities results from a cancellation between stability seen by the nuclear forces with the increasing stability seen by conventional forces. Greater conventional force effectiveness decreases the stability index still further.

Decreasing the effectiveness of conventional forces against weapons increases the nuclear side's and overall stability, which amounts to an upward translation of both stability curves. Thus, if the conventional forces have few precision or deep-strike weapons or if the nuclear weapons are well dispersed, the effect should be stabilizing. The fundamental problem is that stability falls as the nuclear kill probability increases, i.e., as the nuclear forces make a greater contribution. That means that introducing nuclear forces into a conventional engagement or making them more effective is destabilizing because they suppress the conventional side's second strike. Given the great degradation nuclear forces could inflict on conventional forces before they were dispersed, the conventional force commander has a great incentive to disperse them through attacking, which would start the war, or by dispersal, which would reduce the effectiveness of the nuclear forces, giving the nuclear forces an incentive to strike first.

These results roughly model stability concerns in current theaters. For decades, the US has used the threat of use of nuclear weapons in Europe as an offset to NATO's numerical inferiority in conventional forces, linking their use to US strategic nuclear forces to suppress the stability implications. Recently, Russia decided that the qualitative improvement in US forces

necessitated its shift to a greater reliance on nuclear weapons to stabilize the theater. The impact of this shift can be understood by interchanging the roles of the two sides.

Overall, the extension of the nuclear crisis stability formalism to forces that contain both nuclear and conventional elements is quite natural and appears fruitful. It is straightforward to relate conventional maneuver units and capabilities to missile numbers and weapons. The major new elements are the reduced effectiveness of nuclear weapons against dispersed forces and the lower relative effectiveness of conventional weapons against missiles or bases. Both can be treated with the kill probabilities in the strategic missile formalism. For the roughly equal-capability forces treated here, it is shown that high nuclear kill probabilities can be used to offset the effects of greater conventional force dispersion and dispersal, but that such kill probabilities also imply significant reductions in stability unless offset by survivable or out of theater forces.

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Fig. 1. f numerical & analytic vs weapons

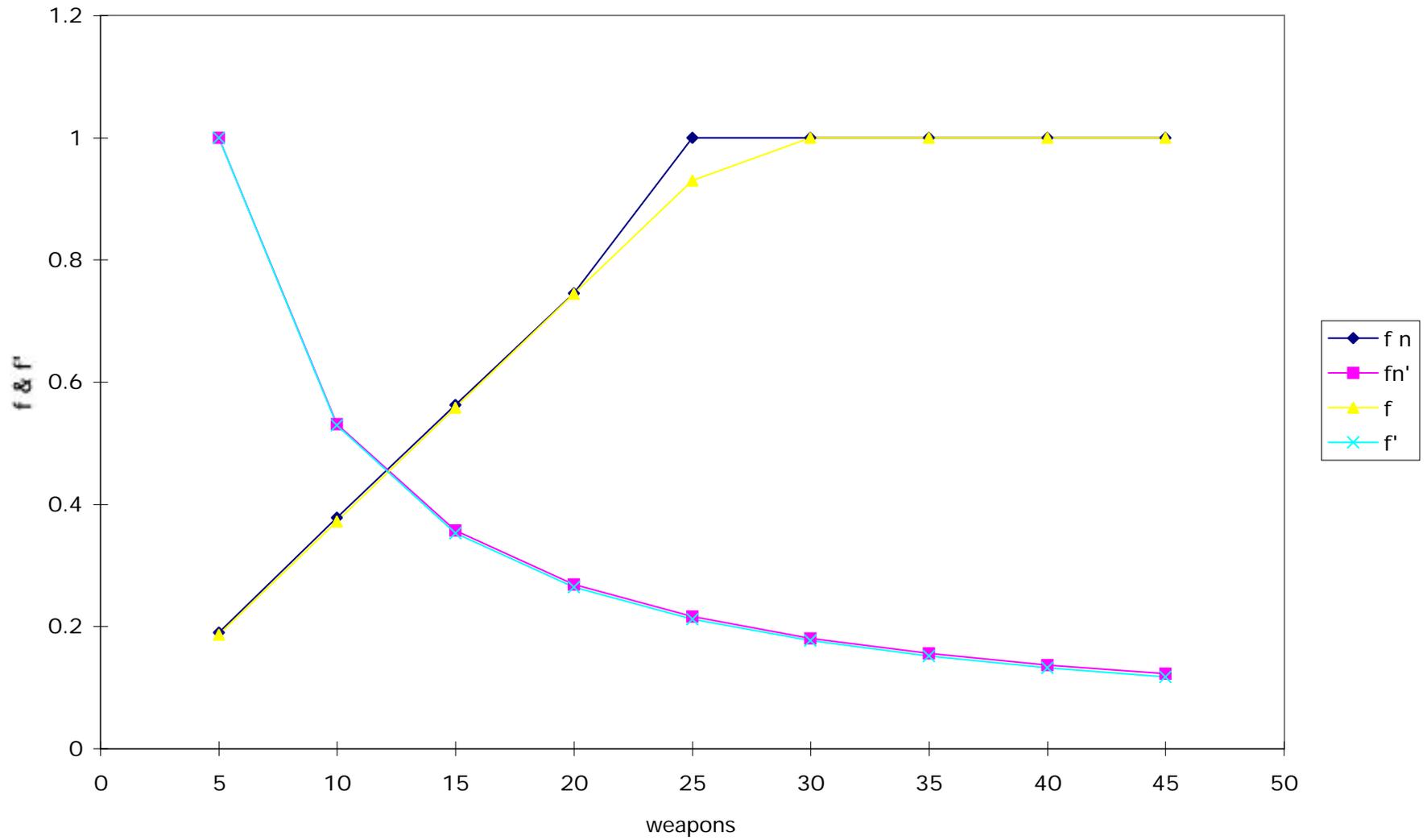


Fig. 2. optimal f v p

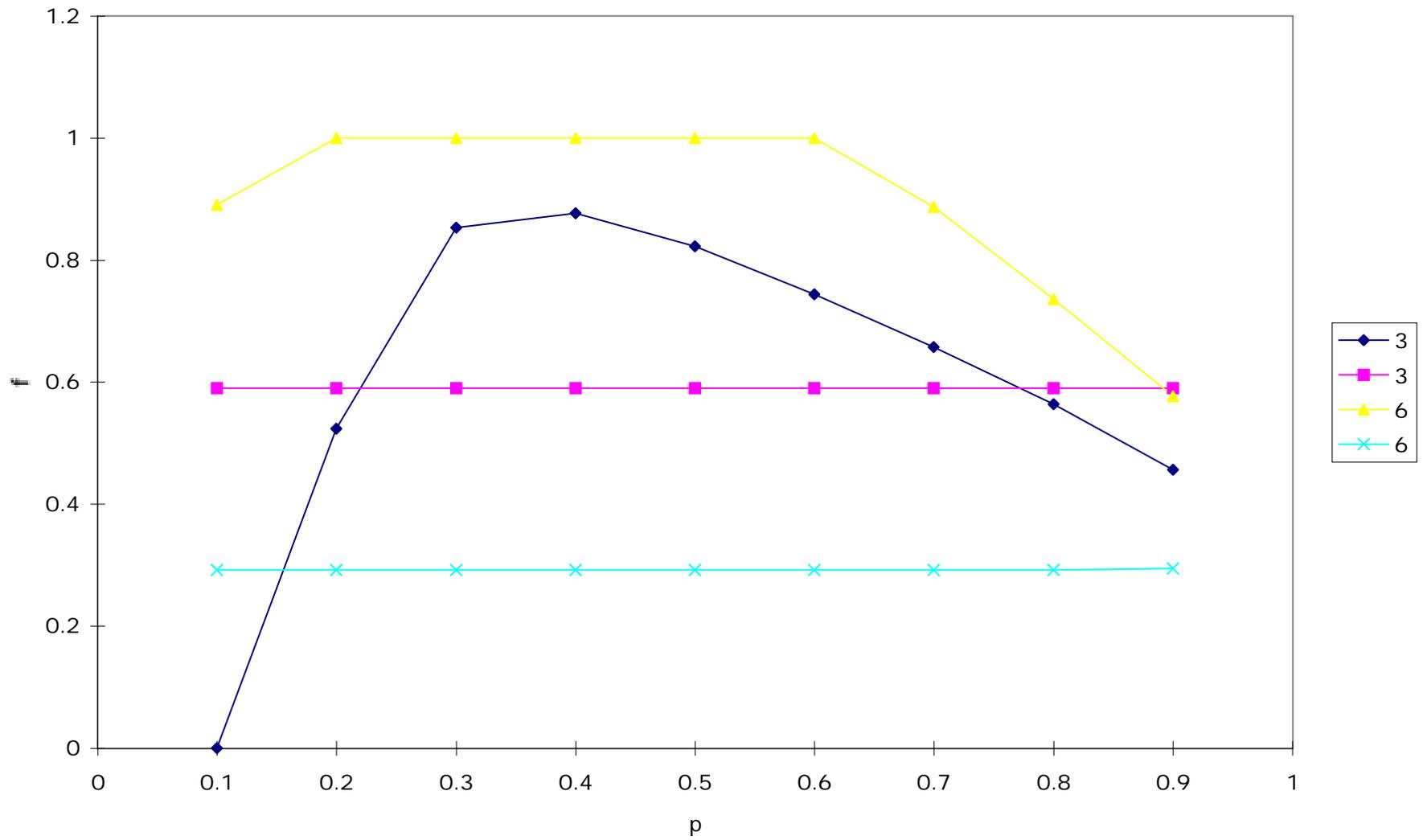


Fig. 3. First strikes on value vs kill probability

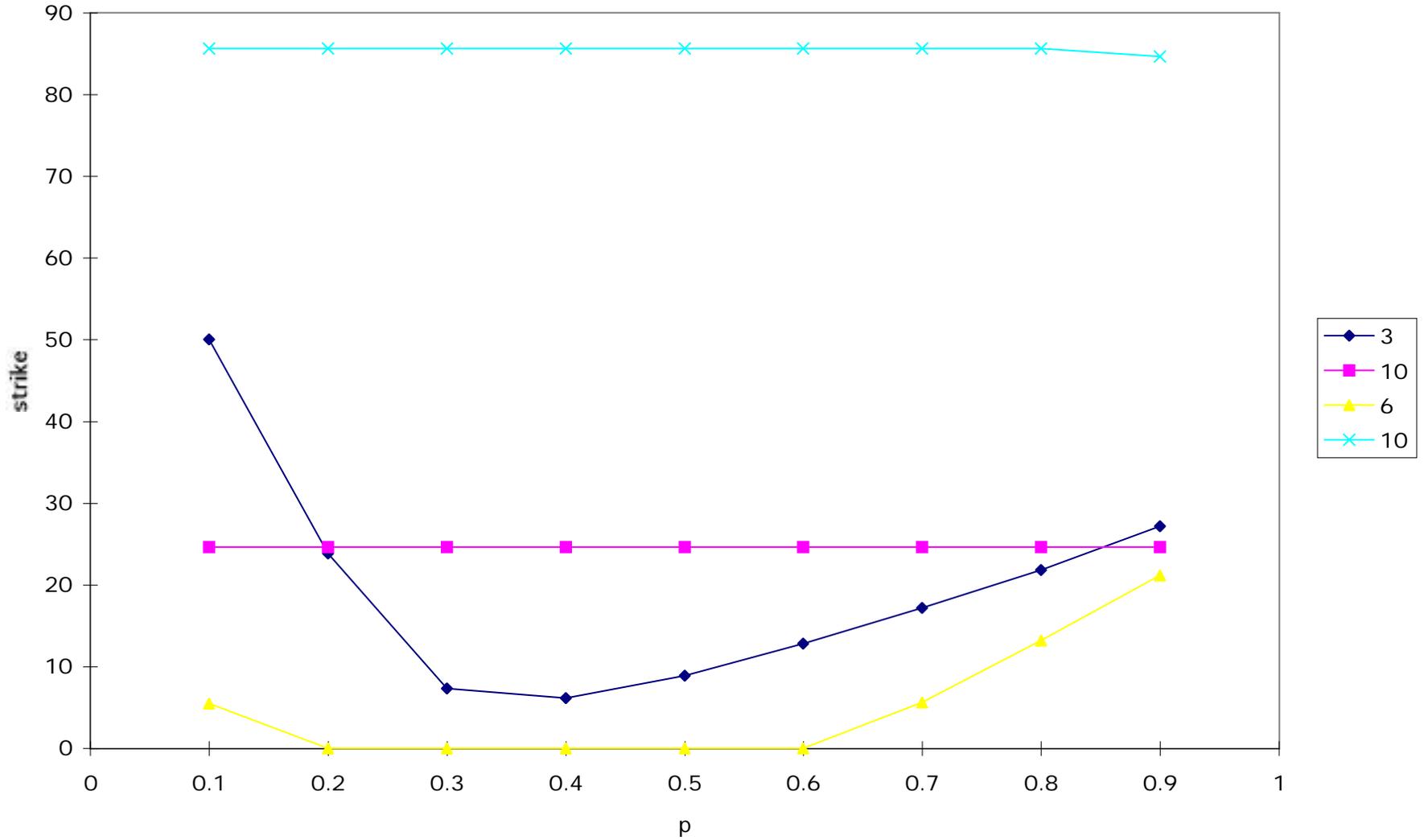


Fig. 4. Survival probability vs p

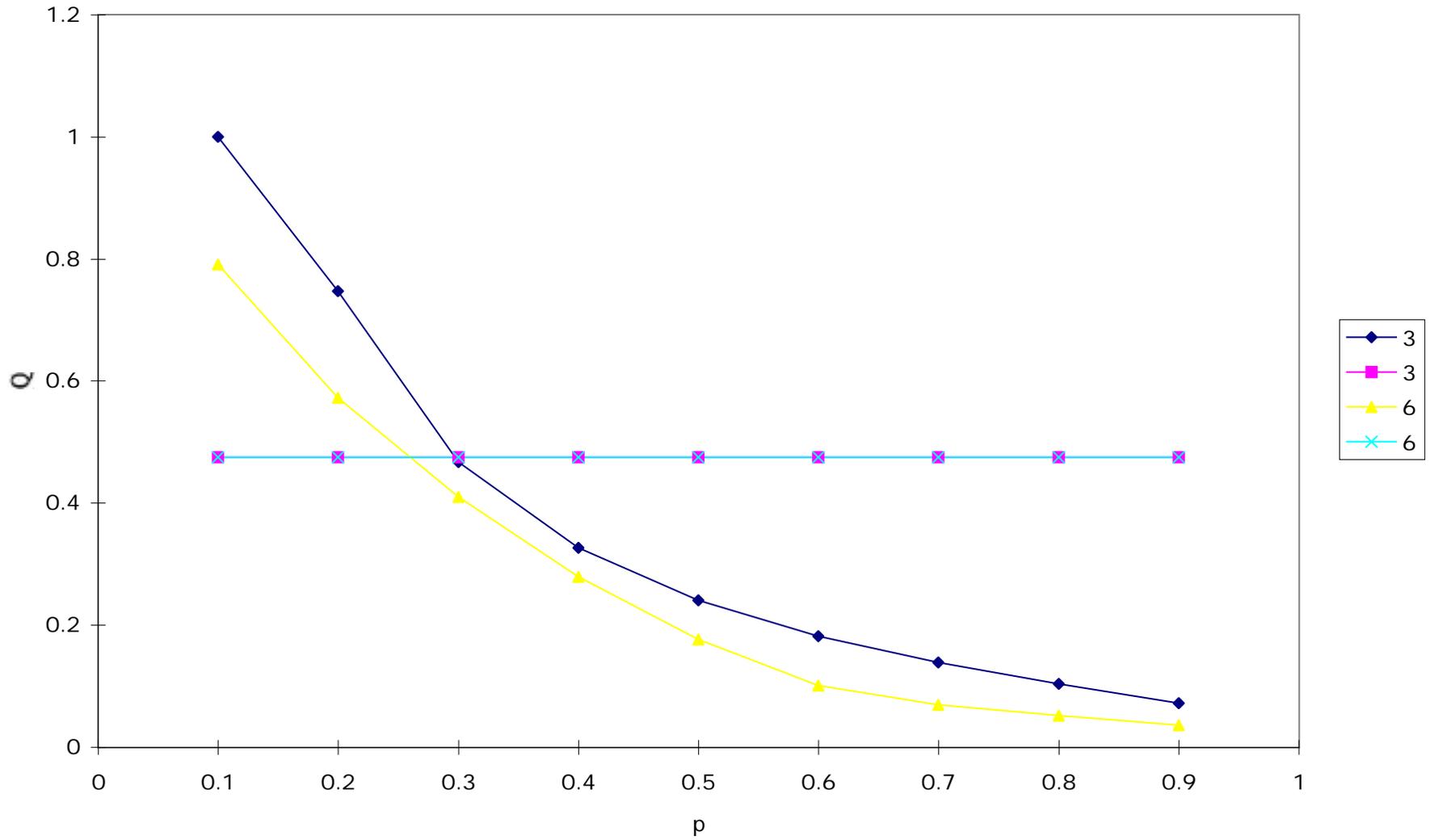


Fig. 5. Second strikes vs p

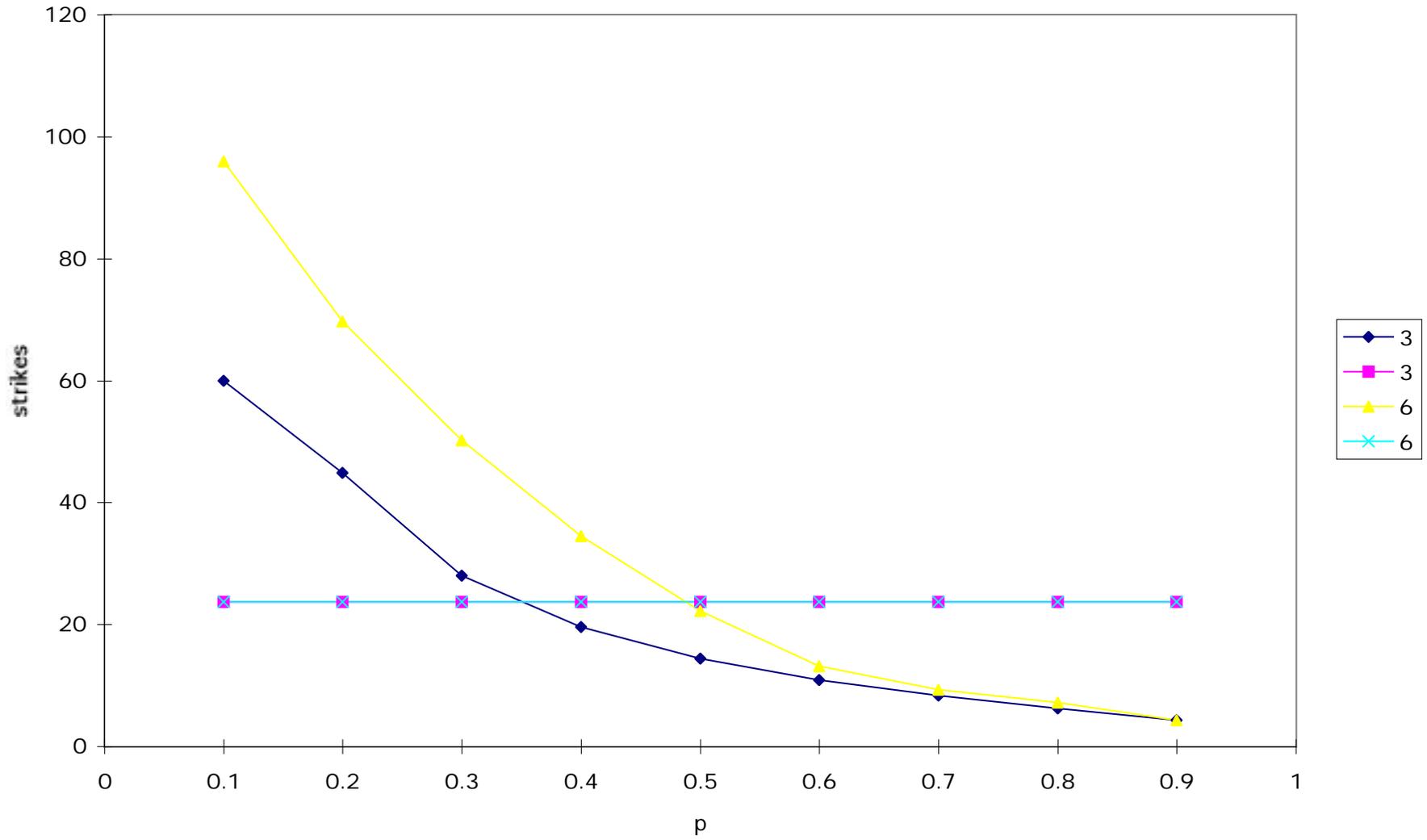


Fig. 6. Strike differences vs p

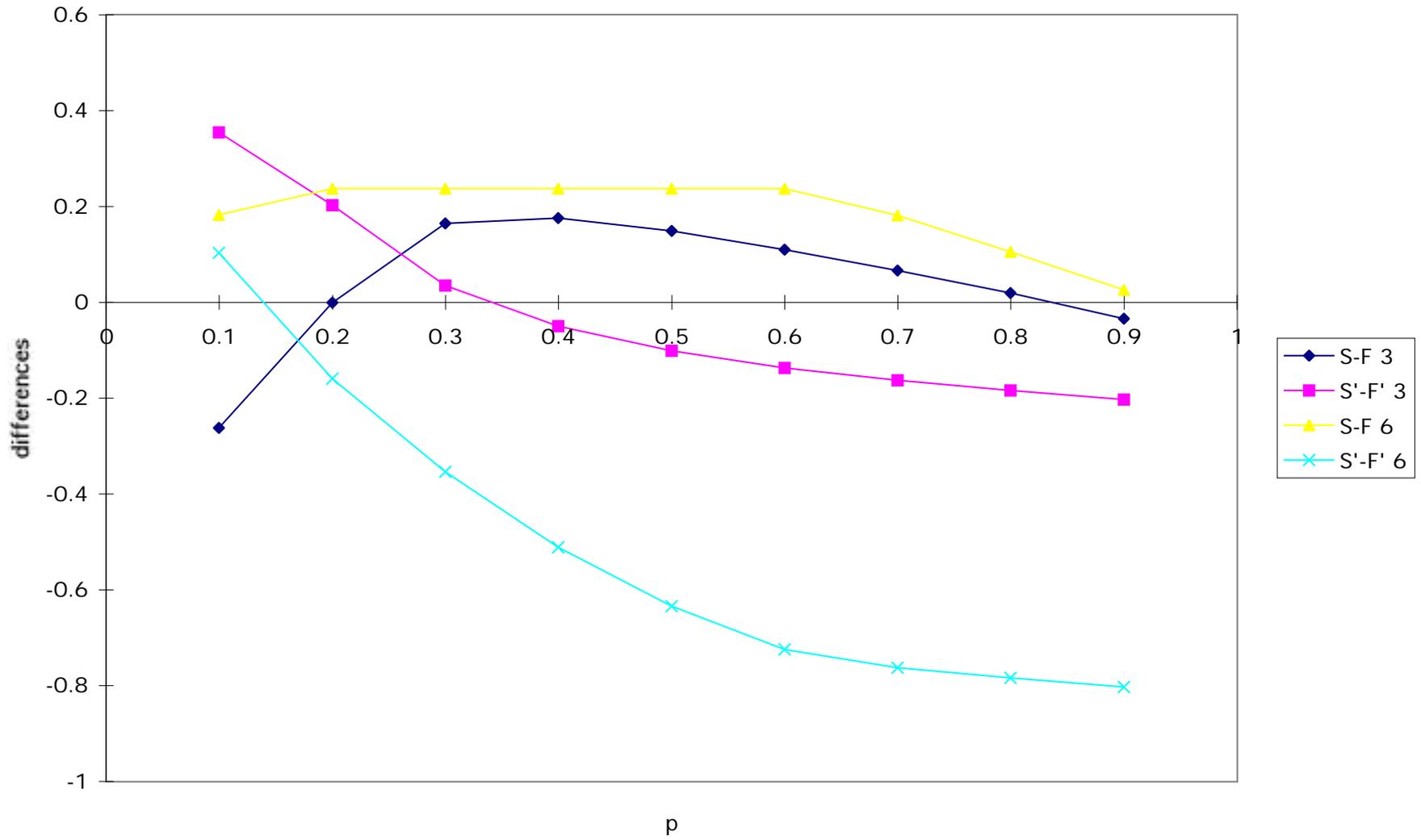


Fig. 7. Stability indices vs p

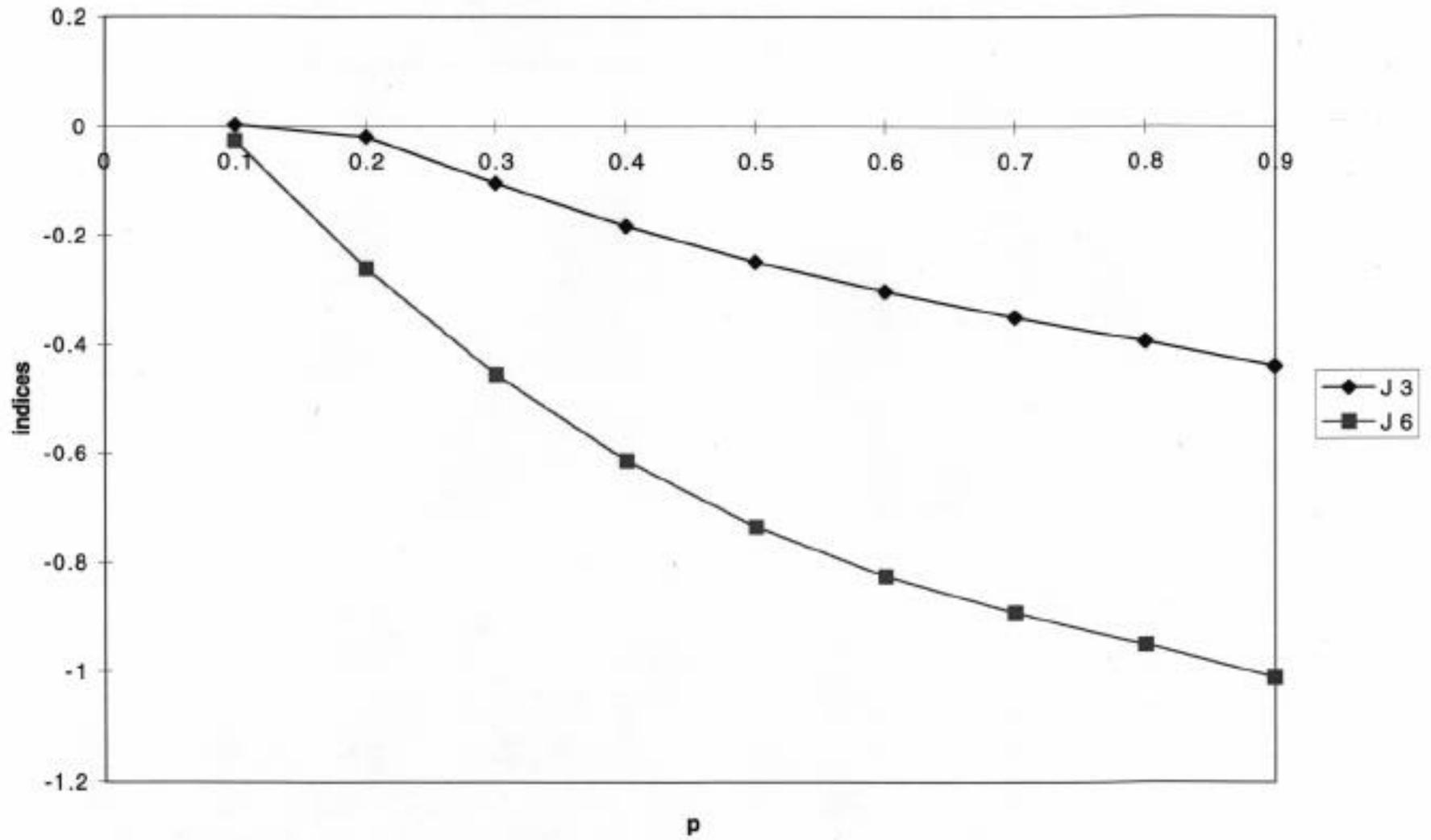


Fig. 8. Stability indices vs p

